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The Distribution of Stress with Depth in the Lithosphere: Thermo-Rheological and Geodynamic Constraints [and Discussion]

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The distribution of stress with depth in the lithosphere: thermo-rheological and geodynamic constraints

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Horizontal tectonic forces arising from the plate-tectonic process give rise to horizontal stress within the lithosphere whose depth distribution is controlled by plastic and brittle strength. A constant force model of lithosphere deformation incorporating the brittle–viscoelastic response of each infinitesimal component of lithosphere allows for the transient computation of lithosphere stress and overcomes the steady-state approximation of the constant strain-rate model. For the constant force model, creep in the lower crust and mantle leads to stress decay in these regions and to stress amplification in the upper lithosphere through stress redistribution.

The lithosphere stress–depth relationship is primarily controlled by rheology which depends on geothermal gradient, crustal composition and thickness. For continental lithosphere, low-strength regions exist within, and at the base of, the crust due to vertical contrasts in composition and therefore rheology. Changes in tectonic force with time give reversals in the sign of stress with depth so that lithosphere has a 'memory' of tectonic force history. Continental lithosphere strength calculated by the constant force model is comparable with estimated levels of intraplate tectonic force.

Horizontal stress within oceanic lithosphere arises due to plate-boundary forces and cooling of oceanic lithosphere. Application of the increasing ridge-push force with age to a cooling thermo-rheological model of oceanic lithosphere predicts compressive horizontal stresses whose magnitude increases with age and which decrease almost linearly with depth. A thermo-rheological model of thermal stress for cooling oceanic lithosphere, incorporating plastic and brittle deformation, predicts horizontal compression in the upper lithosphere above deeper tension. Cooling and ridge push combine to produce a compressive stress field in the ocean basins capable of producing brittle failure.

1. Introduction

The large lateral density inhomogeneities associated with constructive and destructive plate-boundary processes, with isostatically compensated plateau uplifts and intraplate hotspots generate horizontal tectonic force within the lithosphere (Fleitout & Froidevaux 1982, 1983; Bott & Kusznir 1984; Fleitout 1991; Bott 1991). Within intraplate lithosphere this horizontal tectonic force, F_x is distributed vertically as stress to give the local non-lithostatic stress depth distribution such that

$$F_x = \int_0^L \sigma_x \,\mathrm{d}z,$$

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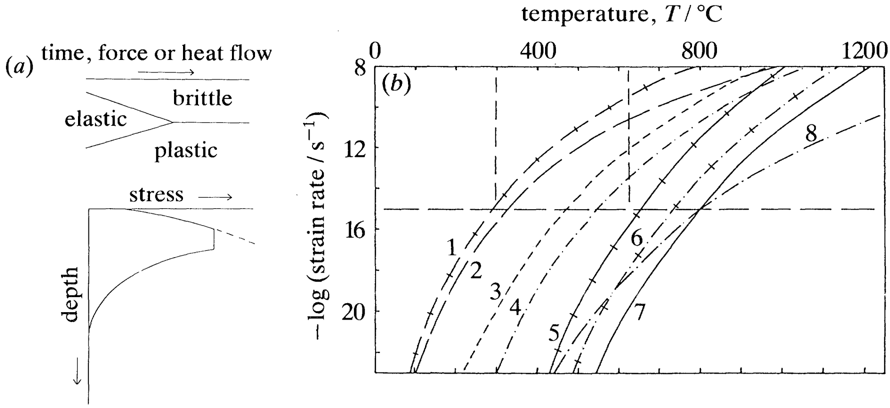

Figure 1. (*a*) Diagrammatic representation of regions of brittle, elastic and ductile deformation within the lithosphere and their control on the stress–depth distribution. (*b*) Strain rate plotted logarithmically against temperature for a range of rocks and minerals relevant to lithosphere deformation. Curves assume a stress difference ($\sigma_1 - \sigma_2$) of 5 MPa and dislocation creep. Data from Ave Lallement (1978), Goetze (1978), Koch *et al.* (1980), Kohlstedt & Goetze (1974), Post (1977), Shelton & Tullis (1981). (1) Wet quartz; (2) dry quartz; (3) anothorite; (4) diopside; (5) wet olivine; (6) wet websterite; (7) dry olivine; (8) dry websterite.

where σ_x is horizontal stress in the x direction, z is depth and L is lithosphere thickness.

Application of stress to the lithosphere results in deformation. Material may deform elastically, by brittle failure and by plastic creep. The distribution of horizontal deviatoric tectonic stress with depth is controlled by the distribution of strength with depth. In the upper lithosphere the dominant non-elastic deformation mechanism is brittle failure. Here strength increases with confining pressure and therefore depth. In the deeper hotter lithosphere the dominant non-elastic deformation mechanism is plasticity. The plastic failure stress envelope decreases with depth due to increase in temperature. The lithospheric stress–depth relationship is shown schematically in figure 1*a*. A region of purely elastic deformation may exist between the regions of brittle and plastic deformation.

The plastic deformation mechanism experiences the greatest variability and therefore exerts the most control on the lithosphere stress–depth relationship. In figure 1*b* estimates of strain rate based on laboratory experiments are plotted against temperature for rocks and minerals relevant to lithospheric plastic deformation. The strain rate curves plotted in figure 1*b* assume a dislocation creep mechanism with a power law stress and temperature dependence for strain rate $\dot{\epsilon}$ given by:

$$\dot{\epsilon} = A \exp(-E/RT)\,\tau^n,$$

where T is temperature, τ deviatoric stress and E activation energy.

Strain rate increases strongly with increase in temperature for all materials. In the mantle plastic deformation is controlled by olivine, while in the crust plastic deformation is controlled by quartz (upper crust) or plagioclase (lower crust). At the same temperature the quartzo-feldspathic crust is substantially weaker than the olivine mantle.

For a geologically significant strain rate of $10^{-15}\ s^{-1}$ a critical temperature of 300 °C must be exceeded for plastic deformation of wet quartz while for olivine a

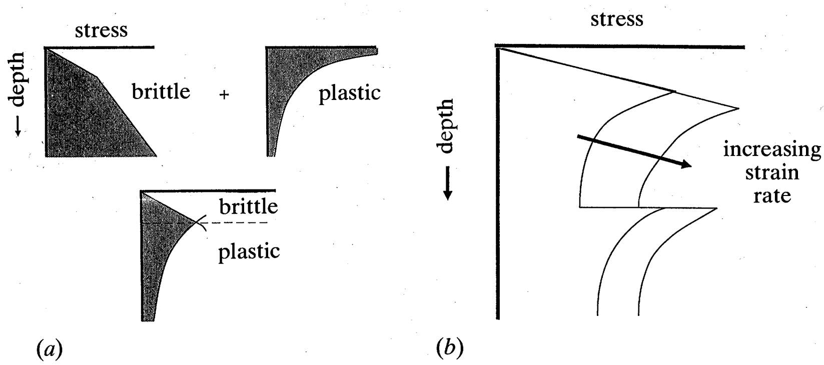

Figure 2. (*a*) Schematic representation of brittle and plastic strength with depth and their control on resultant stress with depth. (*b*) Schematic plot of stress versus depth for the constant strain rate model which assumes steady state deformation. Increasing strain rate increases stress within the plastic failure region and the force required to deform the lithosphere.

temperature of 600–700 °C must be exceeded (Kusznir & Park 1984, 1987). The strong temperature dependence of lithosphere rheology suggests that geothermal gradient will have a dominant control on the stress–depth relationship and therefore on the strength of lithosphere.

2. Constant strain rate and constant force models

(*a*) *Constant strain rate models*

In discussing the distribution of tectonic stress in the lithosphere it is often assumed that the whole of the lithosphere is extending or shortening at a constant strain rate (Chen & Molnar 1983). Within the upper lithosphere the stress–depth distribution follows the brittle failure envelope (figure 2*a*). In the region of plastic deformation beneath the brittle–plastic transition, the horizontal stress must sustain the constant horizontal strain rate with which the lithosphere is deforming and, because temperature increases with depth, the magnitude of the stress decreases with depth. The distribution of stress with depth for a constant strain rate is strongly controlled by strain rate (figure 2*b*). Increasing the horizontal strain rate requires larger values of stress within the plastic deformation region. This allows the region of brittle deformation to penetrate deeper.

The fundamental assumption of constant strain rate, however, is invalid before whole lithosphere failure. Furthermore the horizontal tectonic force required to sustain the deformation is often unrealistically high by several orders of magnitude and substantially greater than that available within the lithosphere.

(*b*) *Constant force models*

An alternative model for the prediction of the distribution of stress with depth is the constant force model (Kusznir 1982; Kusznir & Park 1984, 1987). Fundamental to this model is the assumption that the horizontal tectonic force F_x applied to the lithosphere is constant in time, i.e.

$$F_x = \int_0^L \sigma_x \, dz = \text{const.}$$

and
$$\int_0^L \dot{\sigma}_x \,\mathrm{d}z = 0.$$

Each infinitesimal component of the lithosphere is assumed to behave as a brittle–viscoelastic Maxwell material such that:

$$\epsilon_x = (1/E)(\sigma_x - \sigma_x^0) - (\nu/E)(\sigma_y - \sigma_y^0) - (\nu/E)(\sigma_z - \sigma_z^0) + \epsilon_x^{\mathrm{v}},$$
$$\epsilon_y = (1/E)(\sigma_y - \sigma_y^0) - (\nu/E)(\sigma_x - \sigma_x^0) - (\nu/E)(\sigma_z - \sigma_z^0) + \epsilon_y^{\mathrm{v}},$$
$$\epsilon_z = (1/E)(\sigma_z - \sigma_z^0) - (\nu/E)(\sigma_x - \sigma_x^0) - (\nu/E)(\sigma_y - \sigma_y^0) + \epsilon_z^{\mathrm{v}},$$

where ϵ is total strain, σ is stress, ϵ^{v} is viscous strain, σ^0 is initial stress for brittle failure, E is Young's modulus and ν is Poisson's ratio.

Furthermore each layer of the lithosphere is assumed to be welded together and suffers the same total horizontal strain so that:

$$\mathrm{d}\epsilon_x/\mathrm{d}z = 0.$$

The vertical stress σ_z arising from the application of the horizontal tectonic force is zero since the surface of the solid Earth is a free boundary, i.e. $\sigma_z = 0$.

The assumption of plane strain ($\epsilon_y = 0$) or plane stress ($\sigma_y = 0$) in the horizontal y direction perpendicular to the applied horizontal tectonic force F_x places additional constraints which permit the calculation of σ_x and σ_y in depth and time, t (Kusznir 1982):

$$\sigma_x = \int_0^t \left(\frac{1}{L}\int_0^L k\dot{\epsilon}_\nu \,\mathrm{d}z - k\dot{\epsilon}_\nu \right) \mathrm{d}t' - \frac{1}{L}\int_0^L \sigma_x^0 \,\mathrm{d}z + \sigma_x^0,$$

$$\sigma_y = \int_0^t \left(\nu\dot{\sigma}_x - E\,\frac{(2\sigma_y - \sigma_x)}{6\eta} \right) \mathrm{d}t' + \sigma_y^0 - \nu\sigma_x^0,$$

where k and $\dot{\epsilon}_\nu$ are defined below and η is apparent viscosity.

An alternative assumption to plane strain or plane stress is that both the x and y axes are unconstrained but with $\sigma_x = \sigma_y$ and $\epsilon_x = \epsilon_y$. Solutions for this assumption also exist for σ_x. The assumption that $\epsilon_x = \epsilon_y = 0$ has no physical meaning within the context of the constant force model. The values of k and $\dot{\epsilon}_\nu$ for these assumptions are:

plane strain in y axis:

$$k = E/(1-\nu^2), \quad \dot{\epsilon}_\nu = [\sigma_x(2-\nu) - \sigma_y(1-2\nu)]/6\eta, \quad \dot{\epsilon}_{\mathrm{T}} = (1+\nu)\,\alpha\Delta\dot{T};$$

plane stress in y axis:

$$k = E, \quad \dot{\epsilon}_\nu = \sigma_x/3\eta, \quad \dot{\epsilon}_{\mathrm{T}} = \alpha\Delta\dot{T};$$

unconstrained, y axis $= x$ axis:

$$k = E/(1-\nu), \quad \dot{\epsilon}_\nu = \sigma_x/6\eta, \quad \dot{\epsilon}_{\mathrm{T}} = \alpha\Delta\dot{T};$$

ϵ_{T} is thermal strain and will be used later.

The application of the constant force model to continental lithosphere is shown in figure 3*a*. A compressive horizontal tectonic force of 10^{12} N m^{-1} has been applied to continental lithosphere with a heat flow of 60 mW m^{-2}. Crustal thickness is 35 km. Plastic deformation is assumed to be controlled by wet quartz in the upper crust, by dry quartz in the lower crust and by olivine in the mantle. Horizontal stress is plotted against depth for times of 10^3, 10^4, 10^5 and 10^6 years after the application of tectonic force. The initial elastic stress–depth distribution is shown by a dashed line.

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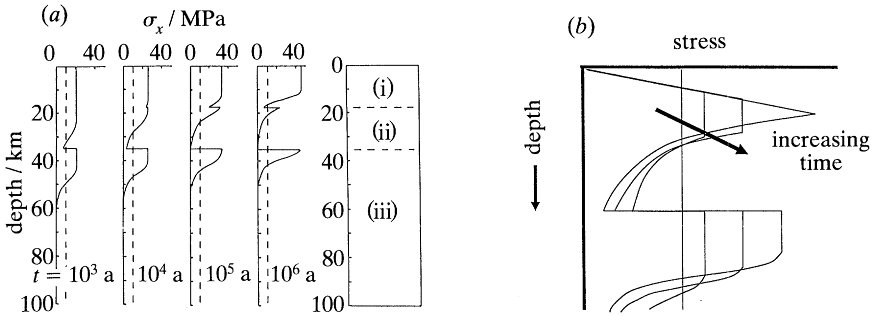

Figure 3. (*a*) Stress against depth at various times after the application of a tectonic force of 10^{12} N m^{-1} for the constant force model of continental lithosphere of average geothermal gradient. No brittle failure. $q = 60$ mW m^{-2}. (i) Upper crust 50% wet quartz; (ii) lower crust 10% dry quartz; (iii) mantle olivine. (*b*) Schematic stress–depth relationship for the constant force model. With increase in time, stress is concentrated and amplified into the upper lithosphere.

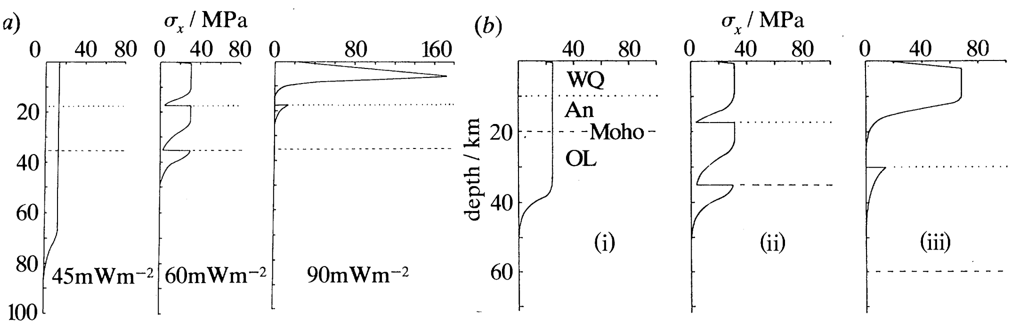

Figure 4. (*a*) Stress plotted against depth for continental lithosphere with cool, average and hot geothermal gradients at 1 Ma after the application of a tensile tectonic force of 10^{12} N m^{-1}. (*b*) Stress against depth for (i) thin (20 km), (ii) average (35 km) and (iii) thick (60 km) crust with average geothermal gradient at 1 Ma.

Plastic deformation in the lower lithosphere causes the decay of stress in the lower lithosphere. Because of force conservation the decayed stress is transferred upwards to increase stress in the upper lithosphere. This process has been called stress amplification by Kusznir & Bott (1977). As time progresses stress decay by creep occurs at higher levels within the mantle and the lower crust. By 1 Ma after the application of tectonic force, stress levels in the upper lithosphere have been amplified by a factor of four.

The behaviour of the constant force model with brittle failure also included is shown schematically in figure 3*b*. In contrast to the constant strain rate model, the constant force model predicts a transient stress distribution with depth and does not assume a constant strain rate with time.

3. Distribution of stress with depth in the continental lithosphere

The geothermal gradient has a dominant control on the lithosphere stress–depth distribution because of the strong temperature dependence of plastic deformation strain rates. In figure 4*a* horizontal stress is plotted against depth for a continental lithosphere with heat flows of 45, 60 and 90 mW m^{-2} subjected to a compressive

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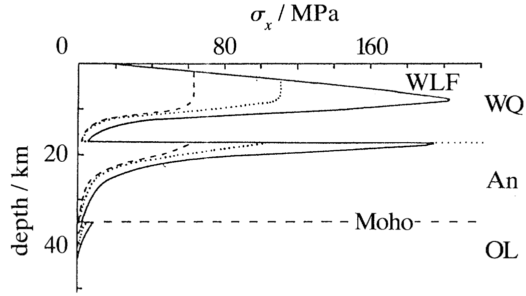

Figure 5. Stress plotted against depth for continental lithosphere for a range of magnitudes of extensional tectonic force after 1 Ma. Heat flow is 70 mW m^{-2}. Increase of the magnitude of the applied force results in WLF. ——, $F_x = 2.5 \times 10^{12}$ N m^{-1}; ·····, $F_x = 1.5 \times 10^{12}$ N m^{-1}; ------, $F_x = 1.0 \times 10^{12}$ N m^{-1}.

tectonic force of 10^{12} N m^{-1}. Wet quartz, plagioclase and olivine rheologies are assumed for the upper crust, lower crust and mantle respectively. Crustal thickness is 35 km. For the cooler lithosphere with low heat flow, stress is carried to greater depth within the lithosphere (75 km for 45 mW m^{-2}). As heat flow increases, creep extends to a higher level in the mantle and lower crust until for 90 mW m^{-2} the stress is concentrated entirely into the upper crust. Substantial brittle failure occurs in the topmost lithosphere for the models with higher heat flows. For the 90 mW m^{-2} model, plastic creep and brittle failure cause stress amplification in the upper lithosphere by a factor of ten. Brittle failure is calculated using Griffith (1924) crack theory as modified by McClintock & Walsh (1962).

Because the quartzo-feldspathic rheology of the crust is much weaker at the same temperature than the olivine mantle, crustal thickness exerts a strong control on the stress–depth distribution of the continental lithosphere. This is plotted in figure 4*b* for crustal thicknesses of 20, 35 and 60 km for a fixed heat flow of 60 mW m^{-2}. For a thin crust, stress is carried to a much greater depth and less stress amplification occurs in the upper lithosphere because of the greater thickness of the stronger olivine rheology. As crustal thickness is increased to 60 km more stress relaxation by creep occurs in the lower crust and more stress is concentrated into the upper crust.

The level of stress amplification and the degree of upper lithosphere brittle failure are also controlled by the magnitude of the applied tectonic force.

Figure 5 shows the effect of increasing the applied extensional tectonic force from 1.0×10^{12} N m^{-1} to 2.5×10^{12} N m^{-1}. As tectonic force is increased there is an increasingly deeper penetration of brittle failure.

4. Whole lithosphere failure and the strength of the continental lithosphere

A typical stress–depth distribution, as predicted by the constant force model, is shown in figure 6*a*. The region of brittle failure in the upper lithosphere is separated from the region of plastic deformation in the lower lithosphere by a region of purely elastic deformation. Stress is maximum within this elastic core. As either the applied tectonic force or the geothermal gradient is increased, stress in the upper lithosphere increases permitting brittle failure to penetrate to a deeper level. Eventually the region of brittle failure and plastic deformation intersect (figure 6*a*) giving rise to whole lithosphere failure (WLF). Only after WLF has occurred can geologically significant strains in excess of a few percent occur (Kusznir & Park 1984).

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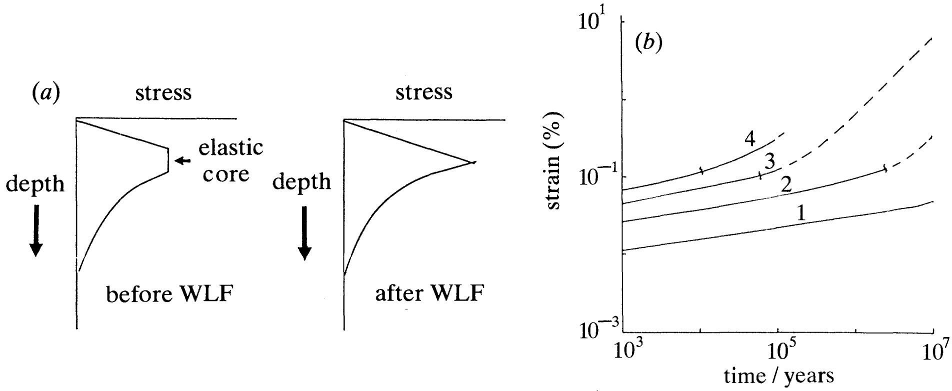

Figure 6. (*a*) Schematic representation of stress with depth for the constant force model showing an elastic core before WLF and its annihilation as WLF occurs. (*b*) Horizontal strain (%) plotted against time for a range of tensile force levels (in units of 10^{12} N m^{-1}) applied to continental lithosphere with heat flow of 70 mW m^{-2}.

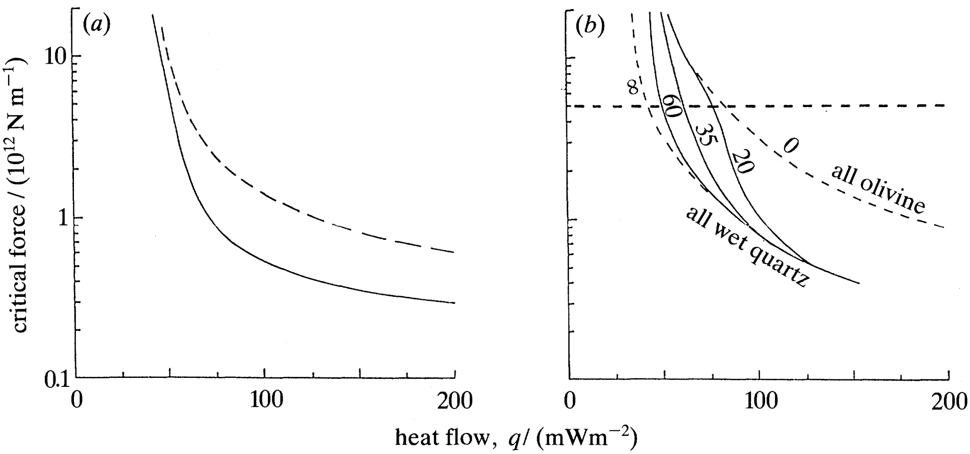

Figure 7. (*a*) Continental lithosphere extensional (——) and compressional (----) strength as a function of heat flow. (*b*) Lithosphere strength for a range of crustal thicknesses (km) with estimated maximum level of extensional tectonic force superimposed. ——, Quartz–olivine lithosphere; ----, end-member lithosphere.

Horizontal strain within the continental lithosphere for the constant force model, is plotted against time in figure 6*b* for different levels of applied extensional tectonic force for a heat flow of 70 mW m^{-2}. For large values of tectonic force WLF is achieved in a relatively short time (e.g. 10^4 years for 4×10^{12} N m^{-1}), while for smaller values a much longer time is required (greater than 10^7 years for 1×10^{12} N m^{-1}). After WLF the stress–depth relationship takes on a steady state and the horizontal strain rate becomes constant. WLF corresponding to a strain rate of 10^{-17} s^{-1} is only achieved after 1000 Ma following application of tectonic force. This implies that for slow constant strain rate models the steady state would never be achieved and that the constant strain rate model is invalid.

In figure 7*a* the force required to generate WLF within 1 Ma is shown as a function of surface heat flow for both extensional and compressional deformation of the lithosphere (Kusznir & Park 1984, 1987). Cool lithosphere in shield areas is inferred

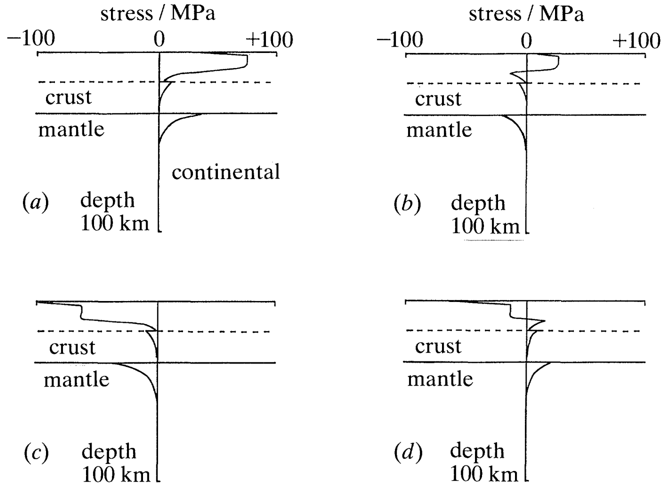

Figure 8. Stress against depth for continental lithosphere subjected to variation of tectonic force with time. See text for detailed explanation. (*b*) and (*d*) have zero net final applied force but non-zero stress.

to be substantially stronger than hot continental lithosphere. For a heat flow of 60 mW m^{-2}, corresponding to the average continental geothermal gradient, the extensional strength is *ca.* 2×10^{12} N m^{-1}. The lithosphere is stronger in compression than tension because of the stronger compressional brittle-failure envelope. In figure 7*b* the extensional strength of the continental lithosphere is shown as a function of heat flow for a range of crustal thicknesses; lithosphere with thick crust is substantially weaker than with thin crust because of the greater proportion of quartzo-feldspathic rheology.

Estimates of the magnitude of tectonic force arising from plate-boundary force and from isostatically compensated loads (Forsyth & Uyeda 1975; Chapple & Tullis 1977; Bott & Kusznir 1984) are:

Plate boundaries	Isostatically compensated loads
ridge push 2–3 $\times 10^{12}$ N m^{-1}	plateau uplift 0–4 $\times 10^{12}$ N m^{-1}
subduction suction 0–3 $\times 10^{12}$ N m^{-1}	continental margins 1–2 $\times 10^{12}$ N m^{-1}
subduction slab pull 0–5 $\times 10^{12}$ N m^{-1}	

The maximum available level of extensional tectonic force is superimposed on the extensional strength of the lithosphere in figure 7*b*. These strength estimates are comparable with the extensional strength for all but the coolest lithosphere in shield areas.

5. Time variation in tectonic force and the stress–depth relationship

Tectonic force within intraplate lithosphere is likely to vary with time as plate-boundary forces change. This has implications for the distribution of stress with depth in the lithosphere. In figure 8*a* the stress–depth distribution is shown 5 Ma after the application of an applied extensional tectonic force of 1×10^{12} N m^{-1} to continental lithosphere with a heat flow of 60 mW m^{-2}. A further incremental compressional force of -1×10^{12} N m^{-1} is applied after 5 Ma and the stress–depth distribution is left to evolve for a further 5 Ma. The result is shown in figure 8*b*. While

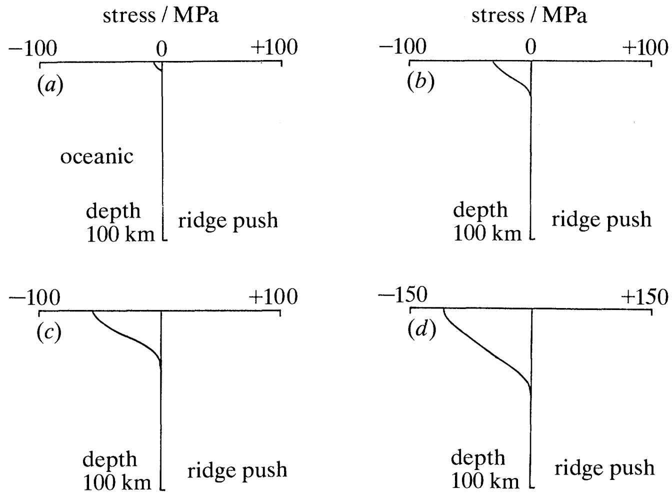

Figure 9. Horizontal stress against depth within the oceanic lithosphere arising from the ridge-push force as predicted by the constant force model. Stress is shown for ages of (*a*) 1 Ma, (*b*) 10 Ma, (*c*) 25 Ma and (*d*) 75 Ma.

the net tectonic force is zero, a complex distribution with depth of both compressive and tensile stress is seen.

The effect of applying a tensile tectonic force of 1×10^{12} N m^{-1} for 5 Ma, followed by an incremental compressive force of 2×10^{12} N m^{-1} for 5 Ma is shown in figure 8*c*. The effect of applying a further incremental tensile force of 1×10^{12} N m^{-1} for a third period of 5 Ma is shown in figure 8*d*. The final tectonic force is zero yet the stress–depth distribution shows both tension and compression. The stress-depth distribution clearly shows a 'memory' for the tectonic force that the lithosphere has experienced. The final models shown in figure 8*b*, *d* show very different stress–depth relationships but have identical zero net tectonic force.

6. Distribution of stress with depth in the oceanic lithosphere

Oceanic lithosphere is subjected to plate-boundary forces, to asthenospheric drag and to thermal cooling and contraction. These processes will result in a complex distribution of stress within the oceanic lithosphere.

If oceanic spreading ridges are assumed to be essentially lithostatic because of the low strength of the hot lithospheric material present there, then the ocean basins will be in absolute horizontal compression. The apparent compressive force responsible for this is called ridge-push force and may be estimated directly from anomalies in geoid height over oceanic spreading ridges (Haxby & Turcotte 1978; Dahlen 1981). The ridge-push force has been estimated to be of the order of 2.8×10^{12} N m^{-1} for lithosphere 80 Ma old. At the ridge crest the ridge-push force is zero. It has been assumed to increase linearly with age at a rate of 0.035×10^{12} N m^{-1} Ma^{-1}. The increasing ridge-push force is applied to a lithosphere which is cooling and becoming mechanically thicker as it ages.

Conservation of horizontal force within the lithosphere gives the equation:

$$\int_0^L \dot{\sigma}_x \, \mathrm{d}x = \dot{F}_{\mathrm{rp}},$$

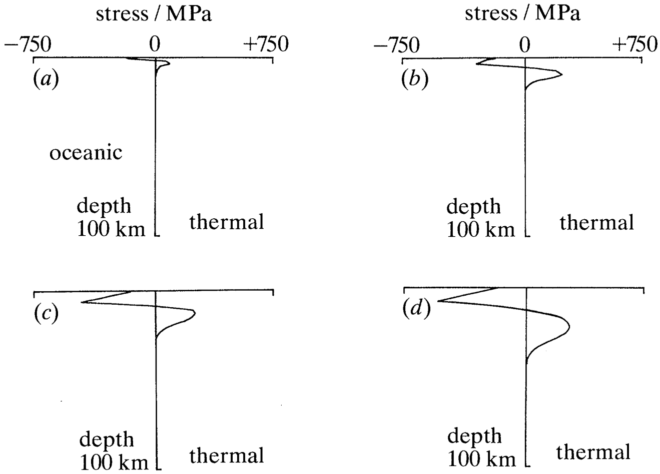

Figure 10. Horizontal thermal stresses within the oceanic lithosphere arising from the cooling of oceanic lithosphere as predicted by the constant force model. Stress is shown for ages of (*a*) 1 Ma, (*b*) 10 Ma, (*c*) 25 Ma and (*d*) 75 Ma.

where $\dot{F}_{\mathrm{rp}}$ is the rate of increase of ridge-push force. Again, the layers of the lithosphere are assumed to be welded together such that $\mathrm{d}\dot{\epsilon}_x/\mathrm{d}z = 0$.

The behaviour of horizontal stress σ_x with depth and time, assuming each infinitesimal component of the lithosphere behaves as a brittle–viscoelastic Maxwell material with power-law stress and temperature-dependent rheology can be shown to be given by:

$$\dot{\sigma}_x = \frac{k}{L}\int_0^L \left(\dot{\epsilon}_\nu + \dot{\epsilon}_{\mathrm{T}} + \frac{\dot{\sigma}_x^0}{k}\right)\mathrm{d}z - k\dot{\epsilon}_\nu - k\dot{\epsilon}_{\mathrm{T}} + \dot{\sigma}_x^0 + \frac{\dot{F}_{\mathrm{rp}}}{L},$$

where ϵ_{t} are thermal strains. k, ϵ_ν and ϵ_{T} have been defined earlier for both plane strain and plane stress in the y direction.

The horizontal stress within the oceanic lithosphere due to the ridge-push force is shown in figure 9 for oceanic lithosphere with ages of 1, 10, 25 and 75 Ma. An olivine rheology in the lithosphere and the thermal model of Parsons & Sclater (1977) have been assumed. As the lithosphere cools it also strengthens and stress is carried to a greater depth. Maximum compressive stress occurs at the surface with an almost linear decrease with depth to zero at about the 650 °C isotherm. For 75 Ma oceanic lithosphere the maximum compressive stress at the surface is about 120 MPa and is insufficient to generate brittle failure.

As the oceanic lithosphere cools as it ages and moves away from the oceanic spreading ridge, it contracts and will generate thermal stresses. The maximum cooling is of the order of 1300 °C and assuming a coefficient of thermal expansion α of 3.5×10^{-5} °C^{-1}, the maximum thermal strains are about 5%. For elastic deformation and a Young's modulus of 10^{11} N m^{-2} the maximum elastic thermal stress would be of the order of 5000 MPa. Clearly such large thermal stresses could not be sustained and would generate brittle and ductile deformation leading to decrease in thermal stress levels.

In figure 10 the result of applying the constant force model to the evolution of thermal stresses within oceanic lithosphere are shown at ocean lithosphere ages of 1, 10, 25 and 75 Ma assuming plane strain in the y direction. The assumption of plane stress in the y direction results in a slightly smaller, but otherwise similar, stress–depth distribution. The horizontal stress within the topmost lithosphere due to thermal contraction is compressive and is underlain by a region of horizontal tension. This is the opposite of a simple intuitive purely elastic model which would give tensile stresses in the upper lithosphere, where cooling has been greatest, above deeper compression. The reason for this fundamental difference is that both brittle and plastic deformation allow the release of the initially tensile stresses in the upper lithosphere created by cooling and contraction, so that when material below cools and contracts it takes the upper lithosphere into compression. The thermal stresses generate substantial brittle failure in compression in the upper lithosphere with some limited tensile brittle failure in the lower part of the mechanically competent lithosphere. After 75 Ma the maximum compressive horizontal stress is of the order of 600 MPa.

The combined effect of ridge-push and thermal-contraction stresses is shown in figure 11*a* for an oceanic lithosphere 75 Ma old. The stress–depth distribution is dominated by the thermal contraction but is biased towards compression due to the compressive ridge-push force. The horizontal stress in the upper lithosphere is again compressive above tension in the lower part of the plate.

Intraplate oceanic seismicity may be used to examine the state of stress of oceanic lithosphere (Wiens & Stein 1983, 1984; Bergman & Solomon 1985; Stein & Pelayo 1991). The distributions of epicentral depth and fault-plane solutions for oceanic intraplate seismicity are shown for comparison in figure 11*b* from work of Wiens & Stein (1983). Epicentral depths follow an isotherm between 600 and 700 °C consistent with the constant force thermo-rheological model and also with estimates of lithosphere thickness obtained from oceanic intraplate loading (Watts *et al.* 1980; Bodine *et al.* 1981). The fault-plane solutions show a tendency for the compressive events to lie in the upper part of the thermo-mechanically competent lithosphere while tensile events lie in the lower part. However, the observational data set of figure 11*b* is heavily weighted in favour of the Indian Ocean which may not be representative due to the anomalously high levels of tectonic force predicted there by Cloetingh & Wortel (1985, 1986).

7. Conclusions

1. Although the constant strain-rate model offers a simple method of calculating the horizontal stress–depth relationship within the lithosphere using brittle and plastic failure envelopes, it makes the assumption of steady-state deformation which is not valid in stable regions.

2. The constant force model incorporates elastic as well as brittle and viscous lithosphere behaviour and accommodates transient lithosphere deformation.

3. The constant force model predicts stress amplification within the upper lithosphere as a consequence of plastic deformation in the lower lithosphere. After stress amplification, levels of stress in the upper lithosphere are sufficient to generate brittle failure within the upper lithosphere.

4. The horizontal stress–depth distribution within the continental lithosphere arising from the application of a tectonic force is strongly dependent on geothermal gradient and crustal thickness.

5. For small horizontal strains, the lithosphere may show an elastic core between the brittle deformation of the topmost lithosphere and the plastic deformation of the middle and lower lithosphere. With increase in time, geothermal gradient or the level of the applied tectonic force, this elastic core may be annihilated as the brittle and

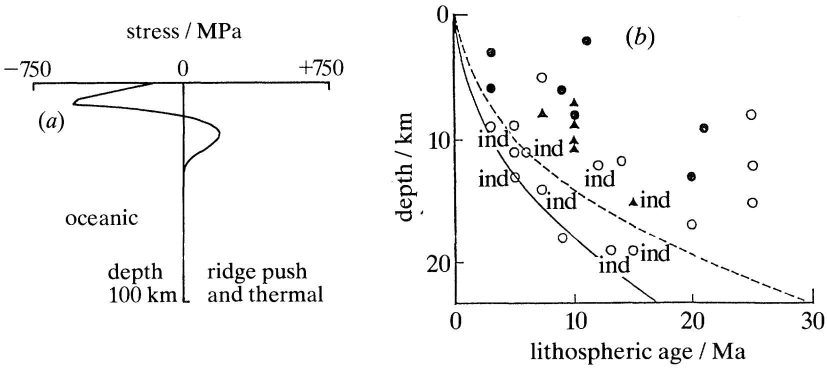

Figure 11. (a) Resultant stress arising from ridge push and thermal cooling for 75 Ma old oceanic lithosphere. (b) Depth and fault-plane solutions for oceanic intraplate seismicity against age (from Wiens & Stein 1983). Maximum depth of seismicity follows an isotherm between 600 and 700 °C. Type of faulting: ○, normal; ▲, strike–slip; ●, thrust.

plastic failure regions intersect and whole lithosphere failure occurs. Geologically significant strains can only occur after WLF has occurred.

6. The strength of the continental lithosphere, as defined by the level of tectonic force required to generate WLF, is strongly controlled by geothermal gradient and crustal thickness.

7. The levels of intraplate tectonic force arising from plate-boundary processes and isostatically compensated loads are comparable with the strength of the lithosphere calculated by the constant force model.

8. The constant force model predicts that the lithosphere stress–depth distribution has a memory of the history of tectonic force so that the stress–depth distribution will not be zero even if the tectonic force applied to the lithosphere is zero.

9. The compressive horizontal stresses within the oceanic lithosphere arising from ridge push are maximum at the surface of the oceanic lithosphere and decrease almost linearly to zero at a depth controlled by the 650 °C isotherm. The depth to which the ridge-push stress is carried by the lithosphere increases with lithosphere age.

10. Thermal horizontal stresses within oceanic lithosphere, as predicted by the thermo-rheological constant force model, are compressive in the upper lithosphere and tensile below. Thermal stresses are sufficient to generate substantial brittle failure.

11. The observed distribution of epicentral depths and fault-plane solutions for intraplate oceanic seismicity is consistent with the prediction of the constant force model which incorporates a combined ridge-push and thermal stress distribution.

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Discussion

C. Vita-Finzi (*University College London, U.K.*). Professor Kusznir is testing the theory by reference to seismic data that assume that deeper events form part of a historical continuum in which the younger events are shallower. Is there some other way of testing the progressive change with time with reference to the oceans?

N. J. Kusznir. The observational data-set required to test the predicted oceanic stress–depth distribution consists of focal depths and fault-plane solutions as a function of lithospheric age. This data are very sparse and those which do exist are heavily weighted towards the Indian Ocean. However, I do not know of an alternative way of estimating the stress–depth distribution in the oceanic lithosphere.

R. B. Whitmarsh (*Institute of Oceanographic Sciences, Wormley, U.K.*). In the continental crust there is a strong/weak/strong sandwich with a weak lower crust. If a new rifted margin is started under tension presumably the lower crust is so weak it cannot transmit this tensional force. Would Professor Kusznir expect the upper crust, therefore, to separate away from the lower continental crust?

N. J. Kusznir. For continental lithosphere with an average geothermal gradient, horizontal tectonic force is carried within the upper crust and topmost mantle, with the lower crust acting as a low-stress–low-strength region. Although the lower crust may be weak, this does not in itself require that the upper crust and upper mantle deform by different amounts or that large-scale horizontal detachments exist within the lower crust.

P. England (*Oxford University, U.K.*). Professor Kusznir concluded that deforming the continental lithosphere at a constant strain rate is reasonable, provided the strain is of large magnitude. What is the time response of the system to changes in stress régime? The figures he showed seemed to depend on the interplay between two time constants. The first is a Maxwell time for the model lithosphere and the second is the rate at which externally applied forces can change. What is the second time constant and what is its size in relation to the Maxwell time constant?

N. J. Kusznir. The constant-force model of the lithosphere does not have a simply defined time-constant for stress decay and transfer. However, for most models the major part of stress variation occurs within 10^5–10^6 years.

Regarding the time-constant variations in tectonic force, I am sure that the amplitude spectrum of tectonic-force variation has finite amplitude at all frequencies. The geological record would suggest that very significant force fluctuations occur over periods of the order of 10^6–10^7 years. If this is so then fluctuations in tectonic force occur with a longer time-constant than the time taken for stress decay and transfer within the lithosphere.

M. H. P. Bott (*Durham University, U.K.*). What happens when a régime changes from tension to compression? What is the memory of the system? I wondered whether the timescale or the disappearance of the memory is of the order of 2 Ma.

N. J. Kusznir. The stress–depth relationship will be dependent on both the level of tensional and compressional force and the time elapsed, and not simply on the final level of tectonic force. The memory of tectonic force is 'encoded' in the stress–depth relationship and exists because of brittle or plastic failure; without brittle or elastic failure there would be no memory. The memory of tectonic force encoded in the stress distribution should not necessarily disappear with time. Only the application of large levels of tectonic force and whole lithospheric failure can destroy the lithosphere's memory of the stress–depth distribution.

R. Govers (*Utrecht University, The Netherlands*). How would the use of steady-state-flow laws for the variable strain rate flow modelling affect the time constants? I did not understand the physics behind the assumption that the vertical gradient of the horizontal strain rates is constant with depth due to the layers being welded together.

N. J. Kusznir. The creep rates used to calculate the stress–depth relationships in this paper assume steady-state deformation for wet and dry quartz, plagioclase and olivine. The mathematical formulation of the constant-force model could use transient creep; however, reliable transient-creep laws have not yet been determined experimentally.

The constant-force model assumes that total horizontal strain is constant with depth (equivalent to the layers of the lithosphere being welded together). Finite-element analysis confirms that this assumption is valid for distances greater than several lithosphere thicknesses away from the point of application of tectonic force (e.g. a plate boundary) for strains not exceeding a few percent.

P. England. Do the reversals of stress mentioned actually occur within a deforming zone because of the presence of tectonic forces which essentially are due to gradients in crustal thickness or gradients in buoyancy forces which do not just go away?

N. J. Kusznir. The prediction of the constant-force model, that reversals of stress with depth are generated by fluctuations in tectonic force, makes no assumptions about the origin of the horizontal tectonic force.

J. Cartwright (*Imperial College London, U.K.*). Has Professor Kusznir modelled the effects of lithosphere-scale discontinuities, such as shear zones inclined at 45°, which are inherently very weak?

N. J. Kusznir. No. The constant-force is a one-dimensional model assuming pure shear only.

S. Murrell (*University College London, U.K.*). The effect of pore pressure is very important both with respect to the level of stress and the depth of seismicity. This question should be addressed experimentally, not only in the laboratory but also in the field, for example by electrical conductivity measurements or by more detailed seismic measurements. I cannot understand how Professor Kusznir could believe that seismicity could occur in a plastically deforming medium. This has not been observed experimentally.

N. J. Kusznir. Pore pressure will have an important control on the level of stress and also on whether lithospheric material lies within the brittle or plastic deformation fields. The stress–depth distributions presented in this paper are calculated assuming zero pore pressure, corresponding to a maximum end-member of lithospheric brittle strength. Increasing pore pressure to hydrostatic (or above) will reduce the brittle-failure strength allowing brittle failure to penetrate to a greater depth within the lithosphere. The question remains as to what the pore pressure is within the middle and lower lithosphere.